Modeling and Simulation

CSE Written Qualifying Exam

Spring 2023

Instructions

- Please answer three of the following four questions. All questions are graded on a scale of 10. If you answer all four, all answers will be graded and the three lowest scores will be used in computing your total.
- Please write clearly and concisely, explain your reasoning, and show all work. Points will be awarded for clarity as well as correctness.

1 Problem 1

The efficiency of any conservative scheme for parallel discrete-event simulation is limited by *lookahead*. a) Explain what lookahead is and why it limits efficiency. b) Give examples of four different real-world simulation problems that illustrate how lookahead affects efficiency. At least one example should show an instance of lookahead enabling abundant parallelism, and at least one example should show an instance of lookahead severely restricting parallelism. For full credit, include an example where lookahead might vary within the same simulation; furthermore, don't just write in general terms, but try to be specific and give illustrative examples. (For instance, to justify that a given simulation will experience lookahead that either limits or enables parallelism, you will need to be specific about the time-scale of lookahead compared to other features of the system being simulated.)

2 Problem 2

Consider an optimistic parallel discrete-event simulator based on the classical TimeWarp algorithm. A critical issue is memory use, which can grow in an unbounded fashion. a) Explain how memory can grow in this way, that is, when and why would memory use grow without bound? b) Describe how to modify TimeWarp so that can "gracefully degrade" as memory grows. That is, suppose the user supplies an *a priori* memory limit; then, rather than crashing the program when the simulation's memory exceeds this limit, your modified

synchronization scheme allows the simulation to recover and proceed, albeit possibly more slowly.

3 Problem 3

a) You have been asked to develop a mechanistic (not purely empirical/data-driven/machine-learning) mathematical model to predict enrollment in an established Georgia Tech course. Specifically, you should develop a model that predicts for semester N the enrollment E_N, using enrollment in one or more prior semesters, such as E_{N-1}, along with any other factors you choose.

   The simplest model would be to use the previous semester's enrollment: E_{N+1}=E_N given some specified E_0. However, such a model neglects many factors that might influence enrollment in later semesters. Write an updated model that takes into account **three** factors that should improve the model's accuracy. For each of the factors you choose, (i) explain in words and in mathematics the factor/effect you are including and (ii) explain what kinds of data would be needed to inform the model (for example, to provide values for any new parameters).

   You should assume that the course is offered online with no maximum enrollment enforced, and that it has been offered continuously for around 5-10 years (so there is some historical information, but not enough to predict reliably using only curve-fitting; hence the desire for a mechanistic model). You also may assume that the enrollment prediction is being generated a few months before the semester (so, not years in advance, but before registration begins).

b) For the model you developed in part (a), explain two assumptions your model makes. For each assumption, describe a way in which that assumption could be eliminated or relaxed. (You may not use any assumptions already listed here in the problem statement.)

c) Next, you have been asked to consult on the development of a similar model, but this time for the on-campus setting, with the goal of suggesting the appropriate classroom capacity for a future semester. In this case, the course has an enrollment cap given by the capacity of the classroom in which it is scheduled. For a model that can recommend when a different classroom (with larger or smaller capacity) should be used, what main factors would you add to the model and how would they inform your model? You are not required to write out a full model in this case, but you should explain the sorts of terms and information that you would seek to add and what data would be needed/could be used to tailor your model to a particular course.

4 Problem 4

Consider the equation

$$\ddot{x} + \alpha x + \beta x^3 = 0, \tag{1}$$

where $\alpha > 0$ and β can be positive or negative. Let $y \equiv \dot{x}$ for the purposes of constructing integral curves and phase portraits.

Consider (1). For $\beta > 0$:

a) Find the singular point(s).

b) Of what type is it (are they)? e.g., center, source, sink, spiral, saddle. If appropriate, is it (are they) stable or unstable?

c) Find the equation(s) for the integral curve(s) (curves of constant "energy").

d) Draw a qualitatively correct phase portrait.

For $\beta < 0$:

e) Find the singular point(s).

f) Of what type is it (are they)? e.g., center, source, sink, spiral, saddle. If appropriate, is it (are they) stable or unstable?

g) Find the equation(s) for the integral curve(s) (curves of constant "energy").

h) Draw a qualitatively correct phase portrait.