CSE Qualifying Exam, Spring 2023: High Performance Computing

Instructions: Please answer three of the following four questions. All questions are graded on a scale of 10. If you answer all four, all answers will be graded and the three lowest scores will be used in computing your total.

1. Consider the standard point-to-point distributed memory communication model (such as the model used in MPI) with latency τ and per-word transfer time μ. Let A, B, C and D be arrays of size n partitioned on p processors in the following manner: i) A and B as per block distribution, ii) C is as per cyclic distribution, and iii) D as per block-cyclic distribution with block size r. For simplicity, assume rp divides n.

   For each of the following operations, describe the communication primitive to be used and compute the corresponding communication cost.

   (a) $B[i] = f(A[i], A[i+1])$ for some function f and $0 \le i < n$. (20%)

   (b) $B[i] = A[n-i]$. (20%)

   (c) $B[i] = \sum_{j=0}^{i} A[i]$. (20%)

   (d) $A[i] = C[i]$. (20%)

   (e) $A[i] = D[i]$. (20%)

2. Consider a mesh of processors with one element per processor. Suppose the elements in each row are sorted in increasing order from left to right and then the elements in each column are sorted in increasing order from top to bottom. Prove that the elements within each row still remain sorted in increasing order from left to right.

3. Let A be a symmetric matrix.

   (a) Prove that A^2 is symmetric. (20%)

   (b) Present a distributed memory parallel algorithm to compute A^2 efficiently using the above observation. (80%)

4. The r-dimensional transposition network consists of $r!$ nodes, one corresponding to each permutation of the integers $1 \ldots r$. Two nodes in the network are connected if and only if the corresponding permutations can be obtained from each other through one transposition operation, i.e., swapping of two elements in the permutation. For example, (2 1 3 4) is connected to (3 1 2 4) because each is obtained from the other by swapping the first and third elements.

   (a) Draw the 3-dimensional transposition network. (20%)

   (b) What is the degree of the r-dimensional transposition network? (20%)

   (c) Determine the diameter of the r-dimensional transposition network. (30%)

   (d) Determine the bisection width of the r-dimensional transposition network. (30%)