Modeling and Simulation

CSE Written Qualifying Exam

Fall 2024

Instructions

- This is a **CLOSED BOOK** exam. No books or notes are allowed.
- Please answer three of the following four questions. All questions are graded on a scale of 10. If you answer all four, all answers will be graded and the three lowest scores will be used in computing your total.
- Please write clearly and concisely, explain your reasoning, and show all work. Points will be awarded for clarity as well as correctness.

1 Problem 1

Feral cat colonies are common in many parts of the United States. While many people appreciate the opportunity to see these independent cats living in outdoor settings, feral cats can cause ecological problems through their predation on birds and other small animals. In addition, feral cats can carry infectious diseases that they can spread to other cats, including housepet cats that are allowed to roam, as well to as other animal species and even humans. For both reasons, it is generally desirable not to let feral cat colonies grow too large.

Two approaches are commonly used to control the size of feral cat populations. The first involves trapping, neutering, and returning cats, so that these cats will no longer be able to reproduce. The second approach involves removing kittens at a young age for socialization with humans and adoption. (Note that relocating adult cats is recommended only if the cats are in immediate danger, because of the trauma it causes the cats to be separated from their territory and the other cats they socialize with.)

In this problem, you will consider how to develop a model that could help to determine an approach for managing a feral cat colony's population.

a) Write a mechanistic model that could be used to model a feral cat colony's population and the potential effects of *both* approaches for managing the population size (trap/neuter/return and removing kittens). There are many ways this could be done and many factors that could be incorporated. Your model will need to have complexity beyond a basic model,

such as exponential or logistic growth, so that it can account for such factors in appropriate ways. Along with writing the mathematical equations for your model, you should explain your model in detail: explain the meanings of the variables used to represent the system, explain the meanings and roles of the parameters that are used, and explain what each term means (what effect it contributes). As an example, a logistic growth model could be explained as having one term that provides exponential-type growth for small populations, and another term that slows growth as the population size increases so that it will approach the carrying capacity. Justify your choices.

b) What types of data could you use to determine approximate values for the parameters of your model?

c) List at least two important assumptions that your model makes. Discuss how reasonable these assumptions are.

d) Policy decisions often need to be evaluated well before the long-term impacts have developed. Discuss how you could use your model to study the long-term effects of each of the two types of population management strategies (trap/neuter/return and kitten removal) without waiting to track the population's size for many years.

2 Problem 2

Consider

$$\dot{x} = -y + ax(x^2 + y^2)$$
$$\dot{y} = x + ay(x^2 + y^2)$$

where a is a real-valued parameter.

a) Show that the linearized system predicts, incorrectly, that the origin is a center for all values of a

b) Determine the origin's actual fixed point stability (stable/unstable) and type for $a > 0$. Consider what the phase portrait might look like to help.

c) Determine the origin's actual fixed point stability (stable/unstable) and type for $a < 0$. Consider what the phase portrait might look like to help.

3 Problem 3

Consider the following system of differential equations:

$$\frac{dx}{dt} = x(3 - x - 2y), \frac{dy}{dt} = y(2 - x - y). \quad (1)$$

a) Is this system linear or nonlinear? Please justify.

b) Find the fixed points of this system. Hint: there are four.

c) Perform a linear stability analysis of the fixed points that you found.

d) Sketch a phase portrait for this system of differential equations.

4 Problem 4

Consider a parallel discrete-event simulator that uses the Time Warp algorithm on a class of applications with the following property: when the computation rolls back and re-executes events, it is often—but not always—the case that the same events (messages) will be scheduled during the re-execution phase as were scheduled in the original execution.

Describe a scheme for Time Warp that can exploit this property to execute more efficiently than simply re-executing. Then give pseudocode for a Time Warp executive that implements your scheme.