Modeling and Simulation

CSE Written Qualifying Exam

Fall 2023

Instructions

- This is a **CLOSED BOOK** exam. No books or notes are allowed.
- Please answer three of the following four questions. All questions are graded on a scale of 10. If you answer all four, all answers will be graded and the three lowest scores will be used in computing your total.
- Please write clearly and concisely, explain your reasoning, and show all work. Points will be awarded for clarity as well as correctness.

1 Problem 1

You have been asked to develop a mechanistic (not purely empirical/data-driven/machine-learning) mathematical model to predict the spread of a new disease affecting deer. The disease is known to be carried by ticks and can be spread to deer when they are bitten by ticks. Although tick bites are the primary transmission route, affected deer occasionally may transmit the disease to other deer through prolonged close physical contact. The disease is serious enough that some (but not all) animals die from it.

a) Write the equations for a mathematical model that can predict the time course of the infection, assuming that data would be available to determine values of parameters used in the model. Explain the meanings of all variables and parameters in your model.

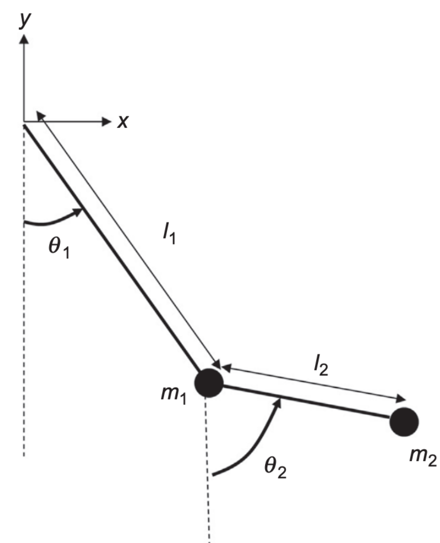
b) For the model you developed in part (a), explain three assumptions encoded in your model (assumptions you made while creating the model that informed the form of the model). You may not use any assumptions already listed in the problem statement.

c) For two of the assumptions you listed in part (b), describe in words and using mathematics how your model may need to be changed if the assumption is incorrect. You should choose assumptions that could require changes in the model. Explain the meaning of the changes and why you made them.

d) Given that the disease is new, it is likely that the values of some parameters in your model are not known with a high degree of certainty. Identify a parameter whose value probably would be uncertain and explain in general how you might account for this uncertainty when making predictions using your model. (In other words: if all parameter values were known with great certainty, you would just solve the model once using those values. What might you do differently if one parameter's value was highly uncertain?)

2 Problem 2

Consider a double pendulum; it looks like this:



where m_i, l_i, and g are constants. The equations of motion for a double pendulum's θ_1 and θ_2 are

$$(m_1 + m_2)l_1\ddot{\theta}_1 + m_2 l_2 \ddot{\theta}_2 \cos(\theta_1 - \theta_2)+ \tag{1}$$

$$m_2 l_2 \dot{\theta}_2^2 \sin(\theta_1 - \theta_2) + g(m_1 + m_2)\sin\theta_1 = 0,$$

$$l_2\ddot{\theta}_2 + l_1\ddot{\theta}_1 \cos(\theta_1 - \theta_2) - l_1\dot{\theta}_1^2 \sin(\theta_1 - \theta_2) + g\sin\theta_2 = 0, \tag{2}$$

where $\theta_{1,2}$ are dependent variables in terms of time (overdots represent derivatives with respect to time).

a) What are the variables $\mathbf{x}$ that make up the equivalent system of equations $\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x})$? Hint: there are four of them.

b) What are the four f_i's? In terms of the constants above and the variables x_i for $i = 1, \ldots, 4$ from a). Show all work.

c) Is your system of equations $\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x})$ linear or nonlinear? Explain.

d) What are the fixed points of your $\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x})$?

e) This system also has four nullclines, one for each x_i. What are they?

f) Computing the fixed points from these nullclines is not possible without a computer algebra system. Still, explain how you *would* do it, ignoring the actual nullclines you computed above.

3 Problem 3

Consider a discrete-event simulation application for studying traffic in a large-scale urban area, such as Los Angeles or Tokyo. Suppose the simulation directly models the road network, where each road segment is a logical process (LP) and events correspond to vehicle activities like arriving at a segment. (That is, the LP for, say, road segment A might process an arrival event with timestamp t for a car that arrives at one end of the road segment. That LP might then schedule a new arrival event for the same vehicle to arrive at a connected road segment B at some future timestamp $t + \Delta t$.)

Suppose this simulation has already been implemented on top of a simulation executive (or simulation engine) that uses the optimistic Time Warp synchronization protocol. Now suppose we wish to model the effects of a sudden unexpected event, like an earthquake: at an arbitrary point in the simulation, the earthquake "occurs" by taking out certain road segments. If this damage occurs at logical simulation time t_e, then any events scheduled or executed by the affected road segment at times $t > t_e$ must be undone.

a) Explain how you would change the simulation application (*not* the underlying simulation executive) to implement this kind of earthquake phenomenon. (We want to see that you understand how optimistic synchronization works by exploiting its properties for this task.)

b) Suppose you are allowed to modify the simulation executive. What would you do differently to accomplish this task?

c) What are the strengths and weaknesses of your two approaches?

4 Problem 4

Consider a distributed simulation of a large number of people who are moving around and communicate using wireless radios. Each person carries a radio, and communication takes place whenever two radios are within some fixed distance (the range of the radio) of each other. Assume each individual is modeled by a distinct logical process (LP). Two LPs exchange messages (events) when the individuals are within radio range of each other. Would it be better to simulate this system using the conservative Chandy/Misra/Bryant protocol or the optimistic Time Warp? Explain your choice giving at least two reasons why.