CSE Qualifying Exam: Modeling and Simulation

Fall 2019

Instructions

- Please answer three of the following four questions. All questions are graded on a scale of 10. If you answer all four, all answers will be graded and the three lowest scores will be used in computing your total.

- Please write clearly and concisely, explain your reasoning, and show all work. Points will be awarded for clarity as well as correctness.

Problem 1

Cellular automata. Consider a 1-D cellular automaton that evolves according to *local rule 110*, which means it maps each possible three bit values to a new bit according to the following table:

	0	1	2	3	4	5	6	7
Current state	000	001	010	011	100	101	110	111
Next state	0	1	1	1	0	1	1	0

(Reading the next-state bits from left-to-right, observe that $2^1 + 2^2 + 2^3 + 2^5 + 2^6 = 110$, hence the name of the rule.)

Furthermore, recall the fact that a NAND (not-and) gate is a *universal* logic operation in that all other logic primitive logic gate operations (and, or, not, exclusive-or) can be constructed from NAND plus constant bits. Using this fact, show that local rule 110 is also universal.

Problem 2

Continuous population modeling. Let $x(t) \geq 0$ be a continuous variable that estimates the size of an insect population at time t, also continuous, and let

$x_0 \equiv x(0)$ be the initial value at time $t = 0$. Suppose that this population changes over time according to

$$\frac{dx}{dt} = r(t)x(t), \tag{1}$$

where $r(t)$ is the rate of growth, which can also vary in time. Observe that when that rate is constant, e.g., $r(t) = \alpha_0$, then the solution is $x(t) = x_0 e^{\alpha_0 t}$, which expresses the idea of exponential growth (when $\alpha_0 > 0$) or decay ($\alpha_0 < 0$).

a. In the *logistic model* of population growth, $r(t)$ is not constant, but rather defined by

$$r(t) \equiv \alpha_0 \left(1 - \frac{x(t)}{\beta_0} \right), \tag{2}$$

where α_0 is a constant growth rate and β_0 is the *carrying capacity* of the population. Explain how one can interpret the logistic model and these constants.

b. An ecologist (someone who studies the relationships of organisms to their natural environments or habitats) has modified the basic logistic model to include an additional term, $p(x)$, defined as

$$p(x) = \frac{x^2}{1 + x^2} \tag{3}$$

$$\frac{dx}{dt} = r(t) - p(x). \tag{4}$$

Explain how this new term changes the model. For full credit, your response should include precise statements based on a detailed analysis of this model.

Problem 3

Simulation model design. You need to develop a strategy to evacuate a large stadium such as Mercedes-Benz stadium (where the Atlanta Falcons play American football) in an emergency situation such as a fire occurring in one portion of the stadium. The metric you are concerned with is the amount of time needed to completely evacuate the stadium. You are to develop a simulation model to evaluate alternate evacuation approaches. Describe the simulation model you would develop to attack this problem. What type of modeling approach would you use (queueing model, cellular automata, differential equations, etc.)? Justify you answer relative to alternate approaches. What are the key state variables and parameters used by the model? What data would you need to collect in order to create a credible model? How would you validate your model, given that performing an actual evacuation is not practical.

Problem 4

Time warp.   One problem with Time Warp is the so-called cascaded rollbacks problem. A cascaded rollback is a situation where one rollback causes a second rollback, which in turn causes a third, etc. Cascaded rollbacks can lead to an excessive amount of rolled back computation and poor performance. (a) Design a variation on the original Time Warp algorithm where rollbacks can occur (when a logical process receives a message in its past) but cascaded rollbacks cannot occur. Describe in detail all changes to the original Time Warp algorithm that are needed to realize this mechanism. Hint: you can eliminate secondary rollbacks if you eliminate the need for anti-messages. (b) Comment on the effectiveness of this approach relative to the original Time Warp algorithm. Specifically, describe the advantages and disadvantages of your approach relative to the original algorithm in terms of performance.