Modeling and Simulation Qualifying Exam — Fall 2020

Answer any 3 questions.

Question 1

A simple dynamical description of a cardiac cell is given by a combination of two equations.

$$D_{n+1} = f(I_n)$$

$$T = D_n + I_n$$

Together, these equations describe how the time duration D of the cell's voltage response to an electrical stimulus depends on the time interval I after the preceding response duration has concluded given a period of electrical stimulation T that is constant.

For values of f that reflect the system's dynamics, there is a fixed point D^* with associated I^* such that $D^* = f(I^*) = f(T - D^*)$.

a. Derive a stability condition for the fixed point for a general function f as well as the period parameter T. Explain your work. You may assume f is differentiable as needed.

b. As the period is varied, it is possible for the stability of the fixed point to change; the point where such a change occurs is called a bifurcation point. Consider the case f is defined as the parabola

$$f(I) = D_0 - \frac{(I - I_0)^2}{\tau_0},$$

where we consider only I values to the left of the vertex so that the function is monotonically increasing. Using the stability criterion you developed in part a, find the value of I at the bifurcation point in terms of D_0, I_0, and τ_0. Then find the values of D and T at the bifurcation point (again in terms of D_0, I_0, and τ_0). Finally, find the specific values of I, D, and T at the bifurcation point for $D_0 = A_0 = 500$, $\tau_0 = 900$ (all in ms).

Question 2

Prove or disprove the following statement: When executing a discrete event simulation program on a parallel computer, the Time Warp algorithm cannot yield an execution time less than the length of the critical path in the event graph. Be sure to state any assumptions you are making in your answer.

Question 3

Consider a simple airport traffic model for a network of airports. Assume each airport has a single runway, and when an airplane arrives, it must queue to use the runway, land, spend time at the gate for boarding, and then departs again. Furthermore, suppose we also wish to model airport closures, that is, sudden airport shutdowns for some random period of time. Give a complete description of a message passing-based parallel discrete event simulation program for this model. (That is, specify the logical processes, events, propose a synchronization scheme, and be sure to state all assumptions and justify your design.)

Question 4

Climate simulations predict that as the earth warms, different regions of the planet will begin to experience a variety of phenomena that make them uninhabitable. For example, increasing temperatures near the equator may make it harder to farm, or rising sea levels may make coastal regions unlivable. One prediction is that humans will begin to move away from such regions into others, which may cause additional problems. For example, people may tend to migrate away from rural farming regions, where agriculture has become difficult, to cities, where migrants will create additional pressure on social programs, transportation, and housing.

To help policy makers understand the impact of human migration due to climate change on cities, you have been asked to lead a team to develop new models and simulations. In particular, your team already has at its disposal two existing simulators: (1) a climate simulator, based on a physics-based continuous-time and continuous-space model, that can predict the average temperature in any region of the world at any moment in time; and (2) a population migration simulator, based on a discrete-time, discrete-space model, that can estimate population levels at the county/province level and migration flows between pairs of counties/provinces. What do you see as the top 3-4 major challenges for your team? Explain what each challenge is, why it is hard, and suggest what strategies you might use to deal with them. You should consider both technical and “non-technical” challenges.