Modeling and Simulation

CSE Written Qualifying Exam

Fall 2021

Instructions

- Please answer three of the following four questions. All questions are graded on a scale of 10. If you answer all four, all answers will be graded and the three lowest scores will be used in computing your total.
- Please write clearly and concisely, explain your reasoning, and show all work. Points will be awarded for clarity as well as correctness.

Problem 1

One approach to modeling a system is to use a set of real devices for some components of the system and have them interact with simulation models of other components. For example, a model for a telecommunication network might include actual network components (routers, traffic generators, etc.) interacting with simulated routers and simulated traffic generators.

Suppose one wishes to use a parallel simulator based on Time Warp for the simulated portion. Describe at least two important challenges that must be overcome to use Time Warp in this fashion, and suggest an approach to overcome each these challenges. Consider a parallel discrete-event simulator that uses the Time Warp algorithm on a class of applications with the following property: when the computation rolls back and re-executes events, it is often—but not always—the case that the same events (messages) will be scheduled during the re-execution phase as were scheduled in the original execution.

Describe a scheme for Time Warp that can exploit this property to execute more efficiently than simply re-executing. Then give pseudocode for a Time Warp executive that implements your scheme.

Problem 2

In the classical description of a parallel discrete-event simulation algorithm (e.g., Chandy/Misra/Bryant, Samadi's method) we describe the algorithm in terms of the behavior of logical processes (LPs). When mapping such algorithms to a parallel computer, the most natural assignment of LPs to processors is one LP per processor. If there are more LPs than processors, then it is still "easy" to do the mapping: we map more than one LP to each processor, and the processor context-switches between its LPs in some fashion (e.g., round-robin). In either scenario, it is theoretically possible to get a speedup of $O(P)$, where P is the number of processors. (Recall that speedup is the sequential execution time divided by the parallel time.)

Suppose the system has *more* processors than LPs. That is, let k be the number of LPs and P the number of processors, where $k < P$. Describe a strategy to map the simulation to the parallel computer in a way that can still (in theory) get a $O(P)$ speedup. If no such strategy can exist, explain why.

Problem 3

1. Mutualism is an inter-species interaction that benefits all species involved.

a) Explain why a model based on the classical Lotka-Volterra equations, such as

$$\frac{dN_1}{dt} = r_1 N_1 + a_1 N_1 N_2 \tag{1}$$
$$\frac{dN_2}{dt} = r_2 N_2 + a_2 N_1 N_2, \tag{2}$$

where the two species are N_1 and N_2 and r_1, r_2, a_1, and a_2 are all positive constants, would be a poor choice for a model of mutualism.

b) Consider the model revision,

$$\frac{dN_1}{dt} = r_1 N_1 \left(1 - \frac{N_1}{K_1} + b_{12}\frac{N_2}{K_1}\right) \tag{3}$$
$$\frac{dN_2}{dt} = r_2 N_2 \left(1 - \frac{N_2}{K_2} + b_{21}\frac{N_1}{K_2}\right), \tag{4}$$

where r_1, r_2, K_1, K_2, b_{12}, b_{21} are all positive constants. Suppose we make the following substitutions: $u_1 = \frac{N_1}{K_1}$, $u_2 = \frac{N_2}{K_2}$, $\tau = r_1 t$, $\rho = \frac{r_2}{r_1}$, $a_{12} = b_{12}\frac{K_2}{K_1}$, and $a_{21} = b_{21}\frac{K_1}{K_2}$. Then we can rewrite the system with fewer parameters (note that we replace τ by t for simplicity):

$$\frac{du_1}{dt} = u_1(1 - u_1 + a_{12}u_2) = f_1(u_1, u_2) \tag{5}$$
$$\frac{du_2}{dt} = \rho u_2(1 - u_2 + a_{21}u_1) = f_2(u_1, u_2). \tag{6}$$

What biologically relevant fixed points (equilibrium values) exist for this system, and under what conditions?

c) For the revised model in part b), assess the stability of all biologically relevant fixed points you found. *Hint:* Use the trace-determinant method for the fixed points with the most complicated expressions (you may use it for the others as well if you like). There is no need to distinguish between nodes and foci/spirals.

d) Based on your findings in part c), what do you expect to happen to the population as $t \to \infty$ in most cases?

Problem 4

Many transportation networks use traffic circles to help manage traffic flow. Traffic circles may range from large ones with many lanes in the circle and multiple lanes of incoming traffic at each point of entry to small ones with only one or two lanes in the circle overall. Managing traffic within the circle can be done in several ways; some examples are listed below. Note that at a stop sign each car is required to come to a complete stop before proceeding, whereas with a yield sign a car must stop if there is oncoming traffic but may proceed if it is safe to do so. Traffic lights allow traffic to proceed freely while the light is green but all traffic must stop when the light is red until it turns green (to allow traffic to flow from a different direction).

- Method 1: Placing a stop or yield sign on every incoming road to give priority to traffic already in the circle.
- Method 2: Placing a yield sign in the circle at each incoming road to give priority to traffic entering the circle.
- Method 3: Placing a traffic light on every incoming road, with a right turn on red prohibited.

Other designs also may be possible, including combinations of these options. Imagine that you have been hired as a consultant for a transportation management company. They have tasked you with developing a model to determine how best to control the flow of traffic in, around, and out of a circle. The model should be general enough to be useful for any circle by changing relevant parameters.

For this problem, you do not need to develop a full-scale model, but you should discuss how you would go about developing a model. What variables and parameters would be needed? What type of mathematical-logical structure might you use for modeling this situation? What assumptions would you make and how would you justify them? What factors do you think would cause the most uncertainty and why, and what are some ways such uncertainty could be mitigated?